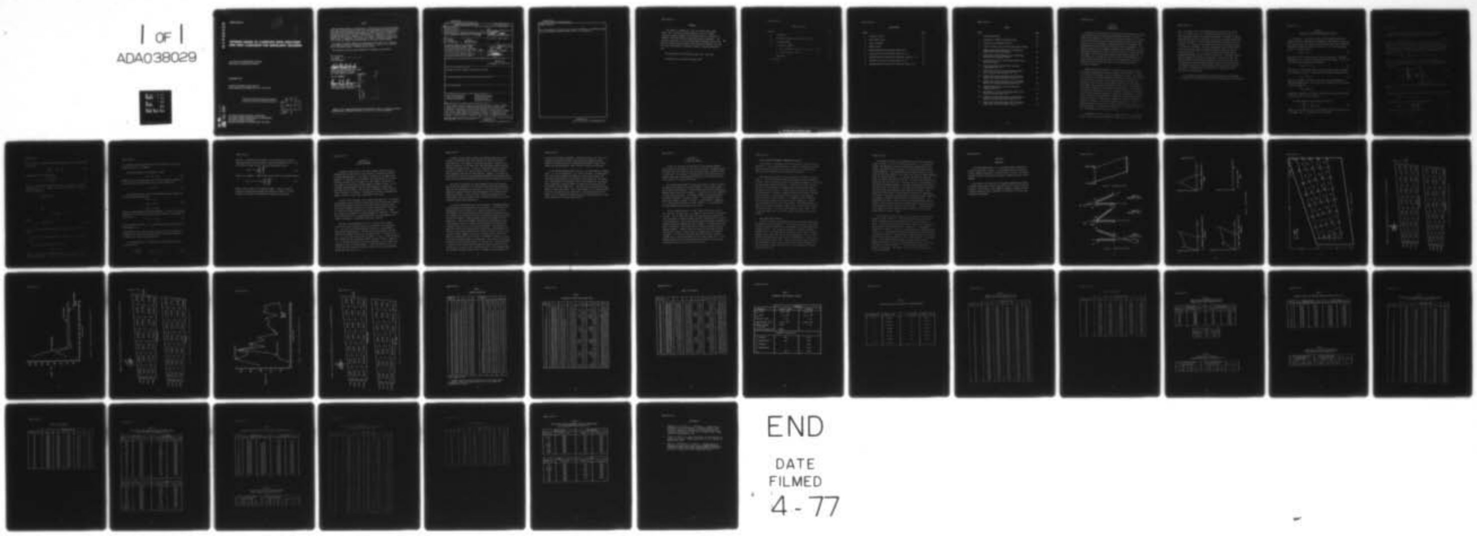
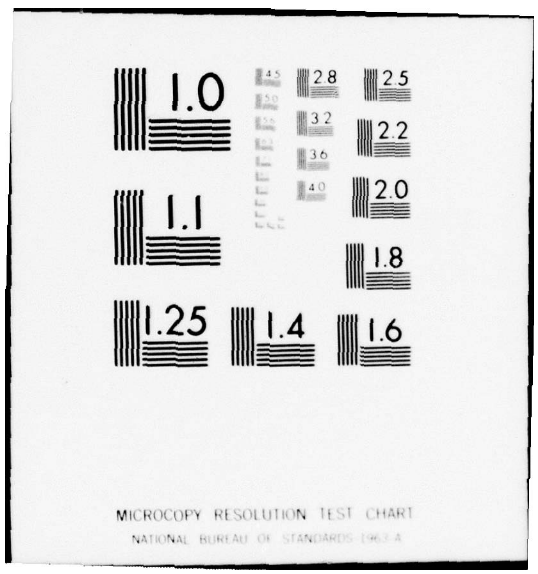
AD-A038 029 AIR FORCE FLIGHT DYNAMICS LAB WRIGHT-PATTERSON AFB OHIO F/G 1/3
OPTIMUM DESIGN OF COMPOSITE WING STRUCTURES WITH TWIST CONSTRAI--ETC(U)
DEC 76 N S KHOT, V B VENKAYYA, L BERKE

UNCLASSIFIED AFFDL-TR-76-117 NL

1 OF 1
ADA038029



END
DATE
FILMED
4-77


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD A 038029

AFFDL-TR-76-117

12

OPTIMUM DESIGN OF COMPOSITE WING STRUCTURES WITH TWIST CONSTRAINT FOR AEROELASTIC TAILORING

ANALYSIS AND OPTIMIZATION BRANCH
STRUCTURAL MECHANICS DIVISION

DECEMBER 1976

TECHNICAL REPORT AFFDL-TR-76-117
FINAL REPORT FOR PERIOD JUNE 1975 - JUNE 1976

Approved for public release; distribution unlimited

AIR FORCE FLIGHT DYNAMICS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

N. S. KHOT
Project Engineer

RICHARD D. KROBUSEK, Maj, USAF
Chief, Analysis & Optimization Branch
Structural Mechanics Division
AF Flight Dynamics Laboratory

FOR THE COMMANDER

HOWARD L. FARMER, Col, USAF
Chief, Structural Mechanics Division
AF Flight Dynamics Laboratory

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

AIR FORCE - 23 MARCH 77 - 300

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFFDL-TR-76-117 (14)	2. GOVT ACCESSION NO.
	3. RECIPIENT'S CATALOG NUMBER (9)
4. TITLE (and Subtitle): OPTIMUM DESIGN OF COMPOSITE WING STRUCTURES WITH TWIST CONSTRAINT FOR AEROELASTIC TAILORING (6)	5. TYPE OF REPORT & PERIOD COVERED: Final Rep[redacted]t June[redacted]975 - Jun[redacted]976
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): N.S./KHOT, L./BERKE, V. B./VENKAYYA, [redacted] K./SCHRADER (10)	8. CONTRACT OR GRANT NUMBER(s) (16)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Design and Analysis Branch (FBR), Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio 45433	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Project - 1467 (17) 02, Task - 146702, Work Unit - 14670246
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio 45433 (12) 46p	12. REPORT DATE: (11) Dec[redacted]976
	13. NUMBER OF PAGES: 39
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Structural Optimization
Finite Element Analysis
Filamentary Composites
Minimum Weight Design
Deflection Constraint
Twist Constraint
Aeroelastic Tailoring

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The objective of this effort was to develop a method to design a minimum weight, fiber reinforced, composite wing structure with a specific twist constraint. Designing a wing for the desired twist is one of the main objectives in the concept of aeroelastic tailoring. The structure is analyzed by a finite element method and the design variables are modified by using a recurrence relation based on an optimality criteria. To demonstrate the method, a wing structure is designed to have a specific negative or positive

012 070

ABSTRACT (cont'd)

twist corresponding to "wash out" and "wash in" conditions. The designs satisfy the twist constraint and the strength criteria in all elements.

FOREWORD

This report is prepared as a part of in-house effort under Project 1467, "Structural Analysis Methods", Task No. 146702, "Structural Analysis Methods for Aerospace Vehicles", and Work Unit 14670246, "Automated Design of Advanced Aerospace Structures". The work was carried out in the Design and Analysis Methods Group of the Analysis and Optimization Branch (FBR), Structural Mechanics Division, Air Force Flight Dynamics Laboratory (AFFDL), Wright-Patterson AFB, Ohio.

The time period of the effort was June 1975 - June 1976.

The manuscript was released in August 1976.

TABLE OF CONTENTS

ILLUSTRATIONS

TABLES

SECTION I
INTRODUCTION

During the last few years the design concept of aeroelastic tailoring aircraft wing structures has gained considerable interest. The basic idea of this concept is to control the anisotropic properties of the wing structure so that the wing will have deformation characteristics under load such as twist and camber which provide beneficial aerodynamic performance while also satisfying strength and flutter requirements. The elastic properties of the composite laminate can be defined by a suitable selection of the number of layers, the fiber direction in each layer and the number of plies in the layer having the same fiber orientations. In this report, it is assumed that the twist of the lifting surface is of primary importance and the structure is to be designed for a specific value of the twist between two points on a chord and satisfy the strength requirements.

Methods developed on the basis of optimality criteria are found to be efficient and have received wide attention in recent years. An overall review of these methods is given in References 1 and 2. The objective in optimization of layered composites treated in this report is to find the thickness of each layer in the composite elements such that the designed structure satisfies the strength criteria in all the elements and achieves the specified displacement pattern as well. The problem of designing a wing type structure with specified twist and displacements is discussed in Reference 3. In this reference, the twist is specified in terms of two unequal displacement constraints at two points on a chord in the structure and the problem is treated as a multiple displacement constraint problem. In the method of Reference 3, the twist constraint could not be separated from the displacement constraints. In the approach discussed in this report, however, the restriction of specifying the displacements is removed and one can specify the required rotation of a line between any two points on a chord.

The planform of a wing structure is shown in Figure 1. A and B are points on the trailing and leading edge respectively, at the tip of the

wing. In Figure 2, AB is the undeflected position of the tip of the wing. In Figure 2(a), A_1 B_1 is the deflected position of the tip for the conventional design. The conventional design is defined as a design that satisfies the stress constraints only and has no specific deflection or twist requirement. In the conventional design the apparent center of rotation C_1 is in front of the leading edge and the tip rotates through Θ_1 ('wash out' condition). Two twist requirements may be defined either (1) to increase the conventional design's wash out twist as shown in Figure 2(b) so that $\Theta_2 > \Theta_1$ or (2) to reverse the direction of the conventional design's twist as shown in Figure 2(c). The second requirement corresponds to the 'wash in' condition. In the first case, the apparent center of rotation moves towards the wing point B, and in the second case the center of rotation moves in back of the trailing edge. In subsequent sections for the convenience of discussion, the 'wash out' and 'wash in' conditions will be referred to as negative twist and positive twist respectively.

In this report, the application of the optimality criteria method is illustrated for the above twist conditions by designing a wing structure.

SECTION II
OPTIMILITY CRITERIA AND RECURRENCE RELATION

The structure is discretized into m finite elements where each element contains n subelements corresponding to the number of layers and each layer consists of a number of plies with the same fiber orientation. The relation between the applied load vector $\{R\}$ and the displacement vector $\{r\}$ can be written as

$$[K]\{r\} = \{R\} \tag{1}$$

where [K] is the stiffness matrix of the total structure. The nodal displacements $\{r\}_i$ of the ith element and the nodal forces $\{f\}_{ij}$ of the jth layer of the ith element are related by

$$[k]_{ij}\ \{r\}_i = \{f\}_{ij} \tag{2}$$

where $[k]_{ij}$ is the stiffness matrix of the jth layer of the ith element. The ith element strains $\{\varepsilon\}_i$ are related to the nodal displacements by

$$\{\varepsilon\}_i = [c]\ \{r\}_i \tag{3}$$

where [c] represents the matrix of strains in the ith element for a unit displacement vector. The stress strain relation for the jth layer of the ith element is

$$[\sigma]_{ij} = [D]_{ij}\{\varepsilon\}_i \tag{4}$$

where [D] is the matrix of elastic constants and $[\sigma]_{ij}$ are the stresses in the jth layer of the ith element.

The total weight of the structure is given by

$$W = \sum_{i=1}^{m} \ell_i \sum_{j=1}^{n} \rho_{ij}\ A_{ij} \tag{5}$$

where ρ_{ij} is the density, A_{ij} is the thickness of the jth layer of the ith element, and ℓ_i is the area of the ith element.

Let the prescribed twist between two points in a structure be denoted by $\bar{c}$. Then the constraint equation can be written as

$$\{r\}^t \{S\} - \bar{c} = 0 \tag{6}$$

where $\{r\}^t$ represents the transpose of the displacement vector and $\{S\}$ represents the virtual load vector. The latter vector consists of forces corresponding to a unit couple applied between the two points where the twist is specified. Using Equations 5 and 6, the Lagrangian $\overline{F}$ can be written as

$$\overline{F} = \sum_{i=1}^{m} \ell_i \sum_{j=1}^{n} \rho_{ij} A_{ij} + \lambda [\{r\}^t \{S\} - \bar{c}] = 0 \tag{7}$$

where λ is the Lagrange multiplier. Differentiating Equation 7 with respect to the design variables A_{ij} gives the necessary conditions for an optimum.

$$\rho_{ij} \ell_i + \lambda \left[\frac{\partial}{\partial A_{ij}} \{r\}^t \{S\} \right] = 0 \tag{8}$$

or

$$\rho_{ij} \ell_i + \lambda \left\{ \left[\frac{\partial}{\partial A_{ij}} \{r\}^t \right] [K] \{s\} \right\} = 0 \tag{9}$$

$$i = 1,2, ---, m$$
$$j = 1,2, ---, n$$

where $\{s\}$ is the displacement vector corresponding to the virtual load vector $\{S\}$.

Differentiating Equation 1 with respect to the design variable A_{ij} gives

$$\frac{\partial K}{\partial A_{ij}} \{r\} + [K] \frac{\partial}{\partial A_{ij}} \{r\} = 0 \tag{10}$$

Multiplying Equation 10 by $[K]^{-1}$ and rearranging gives

$$\frac{\partial}{\partial A_{ij}} \{r\}^t = -\{r\}^t \left[\frac{\partial K}{\partial A_{ij}} \right] [K]^{-1} \tag{11}$$

Substituting Equation 11 in Equation 9 gives

$$\rho_{ij} \ell_i - \lambda \{r\}^t \frac{\partial K}{\partial A_{ij}} \{s\} = 0 \tag{12}$$

$$i = 1,2, ---, m$$
$$j = 1,2, ---, n$$

For the case of a constant strain membrane element, $[k]_{ij}$ is proportional to A_{ij}, thus

$$\frac{\partial K}{\partial A_{ij}} = \frac{1}{A_{ij}} [k]_{ij} \tag{13}$$

Hence, Equation 12 can be written as

$$\rho_{ij} \ell_i - \lambda \frac{\{r\}_i^t [k]_{ij} \{s\}_i}{A_{ij}} = 0 \tag{14}$$

where $\{s\}_i$ are the nodal displacements of the ith element due to the virtual load vector $\{S\}$. Hence, the optimality condition for the twist constraint problem is

$$1 = \lambda \frac{\{r\}_i^t [k]_{ij} \{s\}_i}{A_{ij} \rho_{ij} \ell_i} \tag{15}$$

or

$$1 = \lambda \frac{e_{ij}}{\rho_{ij}} \tag{16}$$

$$i = 1,2, --- m$$
$$j = 1,2, --- n$$

where

$$e_{ij} = \frac{\{r\}_i^t [k]_{ij} \{s\}_i}{A_{ij} \ell_i} \tag{17}$$

e_{ij} is the virtual strain energy density of the jth layer of the ith element.

The optimality condition will now be used to derive a recurrence relation.

The design variable A_{ij} can be written as

$$A_{ij} = \Lambda \alpha_{ij} \tag{18}$$

where Λ is a constant scaling parameter and α_{ij} is the normalized relative design vector. In the case of an isotropic element, j=1 and

A_{i1} represents either the thickness of a plate element or the cross-sectional area of a bar element.

Introducing Equation 18 in Equation 1 yields

$$[K'] \{r'\} = \{R\} \tag{19}$$

where [K'] is the stiffness matrix for the total structure obtained by using the normalized relatives sizes α_{ij} of the elements and

$$\{r'\} = \Lambda \{r\} \tag{20}$$

At the element level, the relations between the actual quantities and the relative quantities are

$$\begin{aligned} \{r'\}_i &= \Lambda \{r\}_i \\ \{s'\}_i &= \Lambda \{s\}_i \\ [k]_{ij} &= \Lambda [k']_{ij} \end{aligned} \tag{21}$$

where the prime denotes the relative quantities. Similarly, the relation between the relative stresses and the actual stresses in an element can be written as

$$\{\sigma'\}_{ij} = \Lambda \{\sigma\}_{ij} \tag{22}$$

where the relative stresses $\{\sigma'\}_{ij}$ are obtained by using the relative displacements $\{r'\}_i$.

From Equation 22 it is seen that the actual stress in the element can be modified to satisfy the selected strength criteria by selecting a suitable value of the scaling parameter Λ.

Introducing Equation 21 in Equation 16, the optimality criteria can be written as

$$1 = \frac{\lambda e'_{ij}}{\Lambda^2 \rho_{ij}} \qquad \begin{matrix} i = 1, - - - m \\ j = 1, - - - n \end{matrix} \tag{23}$$

where e'_{ij} is obtained from Equation 17 by replacing all the actual quantities by their relative values. Multiplying Equation 23 by $\Lambda^2\,\alpha_{ij}^2$ and taking the square root of the resulting equation yields

$$(\alpha_{ij}\Lambda) = B\,\alpha_{ij}\left(\frac{e'_{ij}}{\rho_{ij}}\right)^{1/2} \tag{24}$$

where B is a constant. Equation 24 can be written in an iterative form as

$$(\alpha_{ij}\,\Lambda)_{\nu+1} = B\,(\alpha_{ij})_{\nu}\left(\frac{e'_{ij}}{\rho_{ij}}\right)_{\nu}^{1/2} \tag{25}$$

where $\nu+1$ and ν refer to the iteration number. Since, the design variables are expressed as normalized relative variables, it is not necessary to evaluate the constant B in Equation 25 explicitly.

SECTION III
DESIGN PROCEDURE

A computer program based on the finite element method to design a minimum weight structure with a specified twist constraint was written incorporating Equation 25. Four types of elements are included in the program: (1) constant strain triangles, (2) quadrilaterals constructed from four constant strain triangles, (3) shear panels, and (4) bars. The elements are defined by the node numbers as shown in Figure 3. The triangles and quadrilaterals may be layered composite elements with specified fiber directions. The choice of the number of layers and the fiber orientation is arbitrary. The designer can select any appropriate number of layers and any fiber orientation in each layer.

The basic steps involved in the iterative procedure are: (1) analyze the structure using the relative design vector and find the nodal displacements and the element stresses, (2) find the scaling parameters which satisfy the strength criteria for all elements and the twist constraint, (3) modify the relative design vector by using Equation 25. These basic steps are repeated for a specified number of iterations. The weight of the structure, which is a good indication of the convergence of the iterative procedure, is evaluated during each iteration.

The two scaling parameters are Λ_s for satisfying the strength criteria and Λ_d for satisfying the twist constraint. The design with $A_{ij} = \Lambda_s \alpha_{ij}$ will satisfy the strength criteria for all the elements. Since the scaling parameter is the same for all the elements, for some elements the strength criteria will be oversatisfied. The design with $A_{ij} = \Lambda_d \alpha_{ij}$ will satisfy the twist constraint. If $\Lambda_d \geq \Lambda_s$ then the design will satisfy the twist constraint and the strength criteria. However, if $\Lambda_d < \Lambda_s$, then the design with the scaling parameter Λ_d will violate the strength criteria in certain elements. Since the objective here is to satisfy the twist constraint and the strength criteria, the acceptable designs are those where $\Lambda_d \geq \Lambda_s$.

A design can be scaled to achieve the specified twist only if the structure is twisting in the same direction as the direction of the specified twist. For example, if the unscaled design is twisting in a negative direction, as in Figure 2(b), then the design cannot be scaled to achieve a positive twist. Similarly, the design cannot be scaled to achieve a zero twist, (ie. the two points A and B in Figure 2 moving equal distances after the structure deforms). However, a structure can be designed for zero twist by interpolating between the two designs which twist in the positive and negative directions during consecutive iterations.

The twist is defined as the angle between any two lines each connecting the same two points in a structure before and after deformation. Since the line connecting the two points in a structure moves in a three dimensional space, the correct scaling parameter can be found only by numerical iterations. Direct scaling, as defined by Equation 21 for displacements, can be used only if the line moves parallel to one of the coordinate planes during deformation.

In scaling the designs, one of two procedures is followed depending on the direction of the specified twist constraint. If the wing is to be designed for positive twist, i.e., wash in condition, Λ_d is first calculated for the relative design vector obtained by using Equation 25. Then, based on Λ_d, the relative sizes of the passive elements are changed so that the stresses in these elements satisfy the strength criteria identically. The passive elements include those with negative virtual strain energy in the last iteration and also those which violate the strength criteria with Λ_d as the scaling parameter. The elements with negative virtual strain energy cannot be modified by using Equation 25. They can be resized by selecting their relative thickness such that the stresses in these elements satisfy the strength criteria identically. After this modification, the structure is reanalyzed, a new Λ_d is calculated and the passive members are remodified. This process of correcting Λ_d and modifying the passive members is continued for a specified number of cycles or until the difference between the previously calculated Λ_d and the new Λ_d is smaller than a prescribed value. In the present computer program a maximum of three cycles is allowed for the adjustment of

the sizes of the passive elements. After three cycles, the sizes of all elements with positive virtual strain energy are changed by using Equation 25 and the sizes of all elements with negative virtual strain energy are modified based on their stresses from the last iteration.

In the second procedure where the wing is to be designed for negative twist, i.e. wash out condition, Λ_s is first calculated. In determining this value, those elements with negative virtual strain energy, the shear panels and the bars are not taken into consideration. Then based on Λ_s, the elements with negative virtual strain energy and the elements which violate the strength criteria are modified so that they satisfy the strength criteria identically. The process of modifying the passive members is done three times. The scaling parameter Λ_d is then calculated to satisfy the specific twist constraint. It has been found that the number of cycles required to modify the passive members decreases as the design approaches the minimum weight design.

SECTION IV
ILLUSTRATIVE EXAMPLE

In order to illustrate an application of the proposed method, a wing structure is designed for (1) stress constraint, (2) negative twist constraint (wash out condition) and (3) positive twist constraint (wash in condition). The structure is designed with stress constraint only in order to establish the magnitude and direction of the twist of the tip of the wing for the conventional design.

The finite element model of the wing structure is shown in Figure 4. The nodes of the bottom skin have odd numbers. The structural model has 88 nodes and 158 members. The top and bottom skins are idealized by membrane quadrilaterals and triangles. The spars and ribs are idealized by shear panels and posts. The element numbers and the node numbers defining the elements are given in Table 1. The convention used to define the elements is given in Figure 3. The structure is subjected to a single loading condition as given in Table 2. The coordinates of the node numbers are also given in this table.

The skin elements consist of four layers with fibers in the 0°, 90°, +45° and -45° directions. The number of +45° and -45° plies may not be equal. The 0° fibers are parallel to the direction of the middle spar as defined by connecting nodes 4 and 84 in the top skin or 3 and 83 in the bottom skin. The elastic constants and the allowable strengths for graphite epoxy and aluminum are given in Table 3. The shear panels and posts are aluminum. The maximum stress criteria are used to size the membrane elements. However, only the stresses in the fiber direction and the shear stresses in each layer are checked against the maximum allowables. For the starting design the relative sizes of the bar elements are 1.0 and the thicknesses of the plate element are 0.1. The membrane element for the starting design are assumed to have an equal percentage of plies in the four fiber directions.

1. STRESS CONSTRAINT PROBLEM (CONVENTIONAL DESIGN)

The structure is designed by using the recurrence relation given in Reference 3. This recurrence relation is similar to Equation 25, except that the relative virtual strain energy term e'_{ij} is replaced by the relative strain energy due to the applied set of loads.

The iteration history for the stress constraint problem is given in Table 4. The initial weight of the structure with an equal percentage of plies in the four fiber directions in the membrane elements is 312.84 lbs. The least weight design of 45.45 lbs is obtained in the 14th iteration. The sizes of the elements corresponding to the minimum weight design are given in Tables 5 and 6. In these tables the quantities represent the thicknesses for the plate elements and the cross-sectional areas for the bar elements. Figure 5 shows the distribution of the number of plies in the four fiber directions. The number of plies is obtained by dividing the thickness of the layer by the thickness of a single ply and rounding it to the next higher number. The deflections for the final stress constraint design at nodes 7 and 9 at the tip of the wing are given in Table 7. The tip twists through an angle -5.458°. This is the angle between the lines joining nodes 9 and 7 before and after loading.

2. TWIST CONSTRAINT PROBLEMS

The wing structure is designed for two twist constraint conditions. The purpose of the two examples is to show the application of the twist constraint algorithm (Equation 25) in designing a structure with positive or negative twist. The two angles selected for the twist constraint problems are -7° (wash out) and +2° (wash in). For the twist constraint problem, recall that there are two scaling parameters Λ_s and Λ_d. The designs corresponding to Λ_s and Λ_d will be referred to as 'stress design' and 'twist design' respectively. In the stress design, the stresses in all the elements are less than or equal to the maximum allowable stress and in the twist design, the twist of the tip of the wing is equal to the prescribed value.

The iteration history for the negative twist constraint problem is given in Table 8. For the first three iterations $\Lambda_d < \Lambda_s$, hence the weights corresponding to the stress design are larger than the twist design. The weight of the structure is directly proportional to the scaling parameter. The absolute value of the twist of the tip of the wing for the stress design is less than 7° for the first three iterations. After the fourth iteration $\Lambda_d > \Lambda_s$ and thus the weight of the twist design is greater than the stress design. For these iterations the twist design has the tip rotation of -7° and the stresses in the elements are less than the maximum allowables. The least weight design of 71.86 lbs is obtained at the 11th iteration. The weights for the stress design and the twist design are plotted for each iteration in Figure 6. This figure shows that after the fourth iteration, there is no significant difference in the weight of the structure satisfying the twist constraint. The design has reached a stable position in the design space. The displacements at nodes 7 and 9 for the least weight design are given in Table 9. The thicknesses and cross-sectional areas of members for the minimum weight design are given in Tables 10 and 11. The distribution of the number of plies in the four fiber directions for the minimum weight design is given in Figure 7.

For the positive twist constraint of +2° (wash in), the iteration history is given in Table 12. The twist of the tip for the initial design is negative, and this design cannot be scaled to achieve +2° twist. From the 2nd to the 8th iteration the twist of the stress design increases with each iteration until it reaches +2°. In subsequent iterations for nearly all cases, the twist designs are acceptable designs since $\Lambda_d \geq \Lambda_s$. The weight of the structure versus the iteration numbers is plotted in Figure 8. Figure 8 shows that the weight of the structure oscillates and the design does not reach a stable position in the design space. For the twenty-one iterations, the least weight design is obtained in the last iteration. The displacements at nodes 7 and 9 for this design are given in Table 13. Tables 14 and 15 contain the details of the design. The number of plies in the top and bottom skin is given in Figure 9.

SECTION V

CONCLUSION

It has been demonstrated that an algorithm based on optimality criteria can be used to design a minimum weight wing structure with specific negative (wash out) or positive (wash in) twist. The design satisfies the twist constraint and specific strength criteria in each element.

Future work in this area will be devoted to developing a method to design a wing type structure with specified twist at more than one location. Further work is also needed to develop a program to design a wing structure for twist, strength, and stiffness for flutter and divergence requirements.

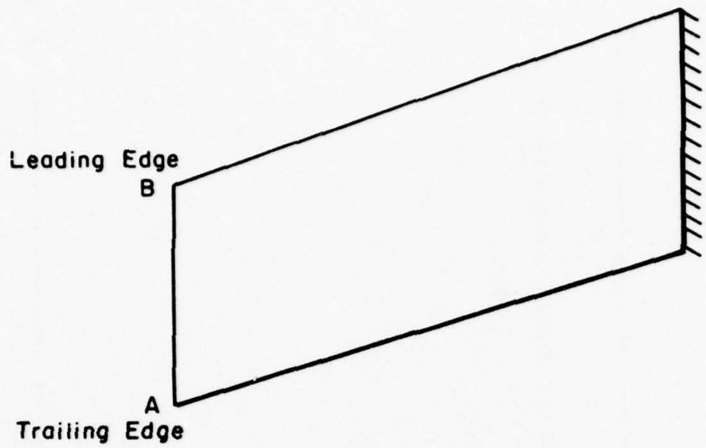


Figure 1. Planform of Wing

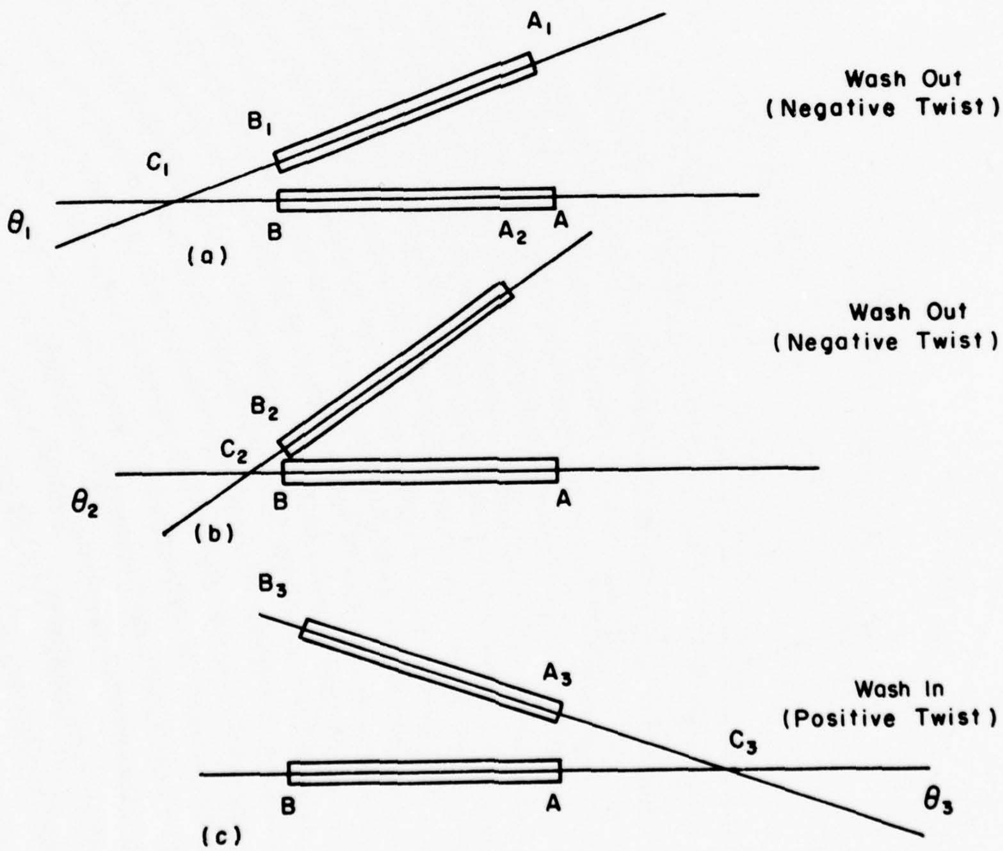


Figure 2. Definition of Twist

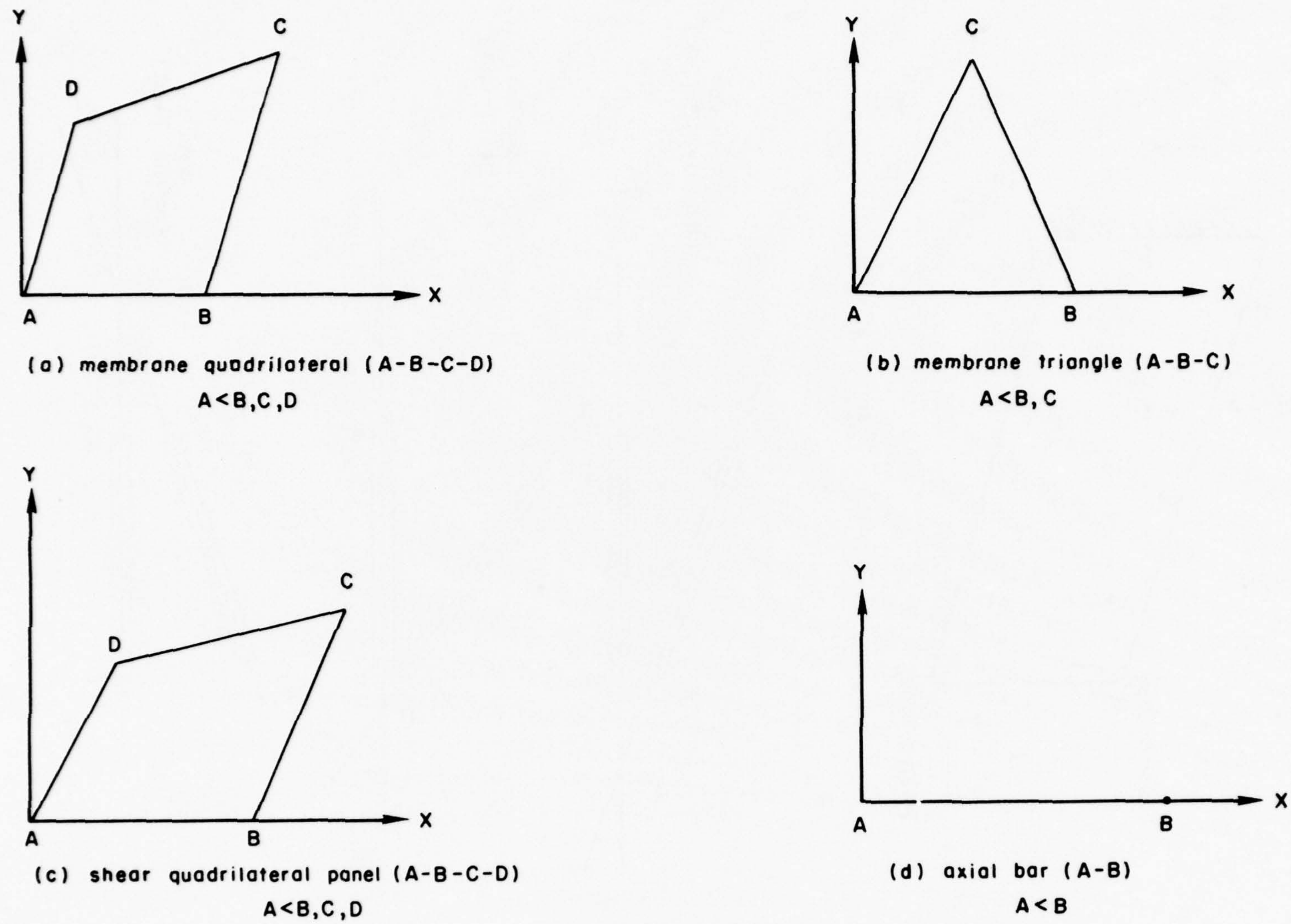


Figure 3. Type of Elements

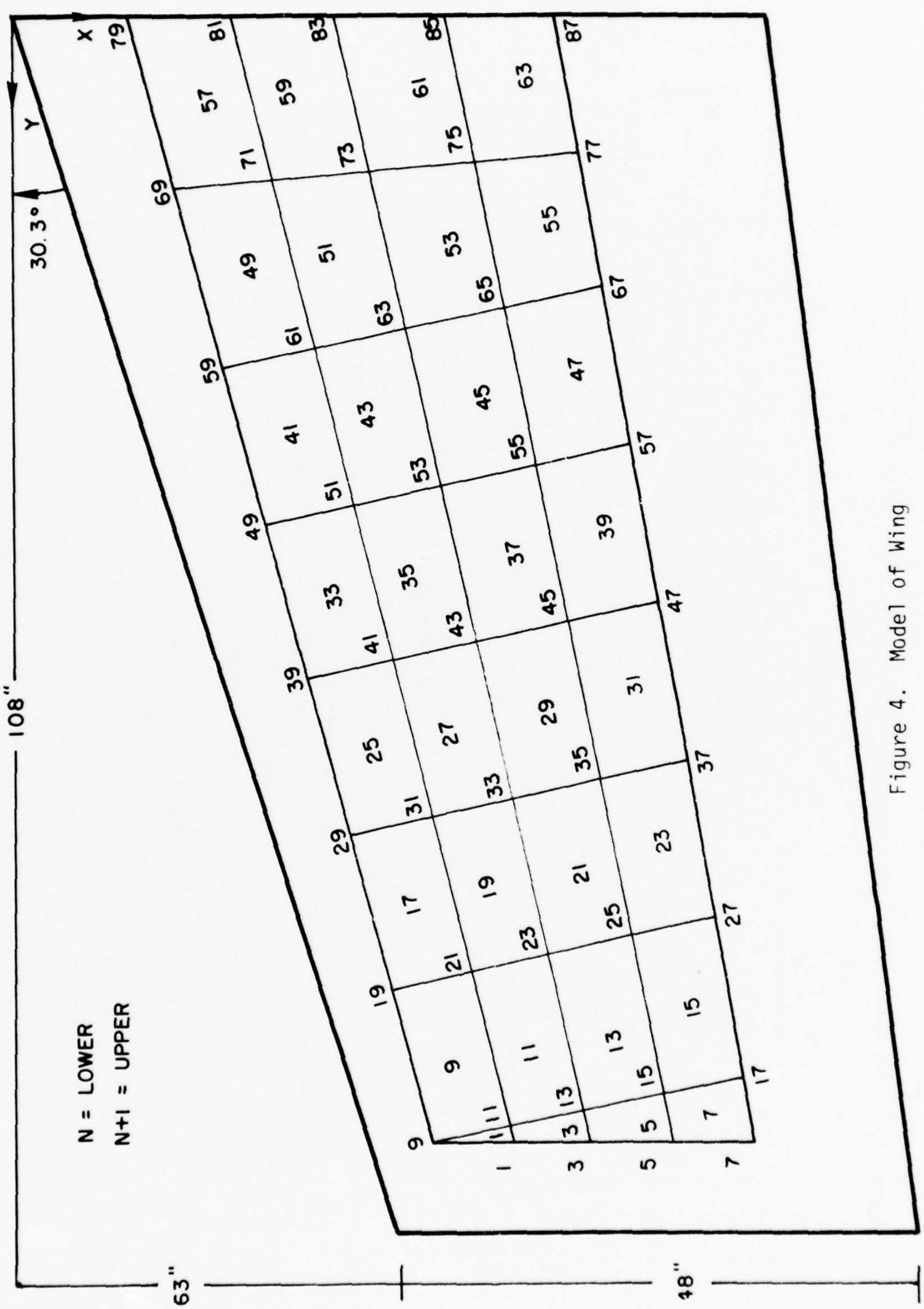


Figure 4. Model of Wing

NUMBER OF LAYERS WITH FIBER ORIENTATIONS IN 0°, 90°, +45°, -45° DIRECTION RESPECTIVELY

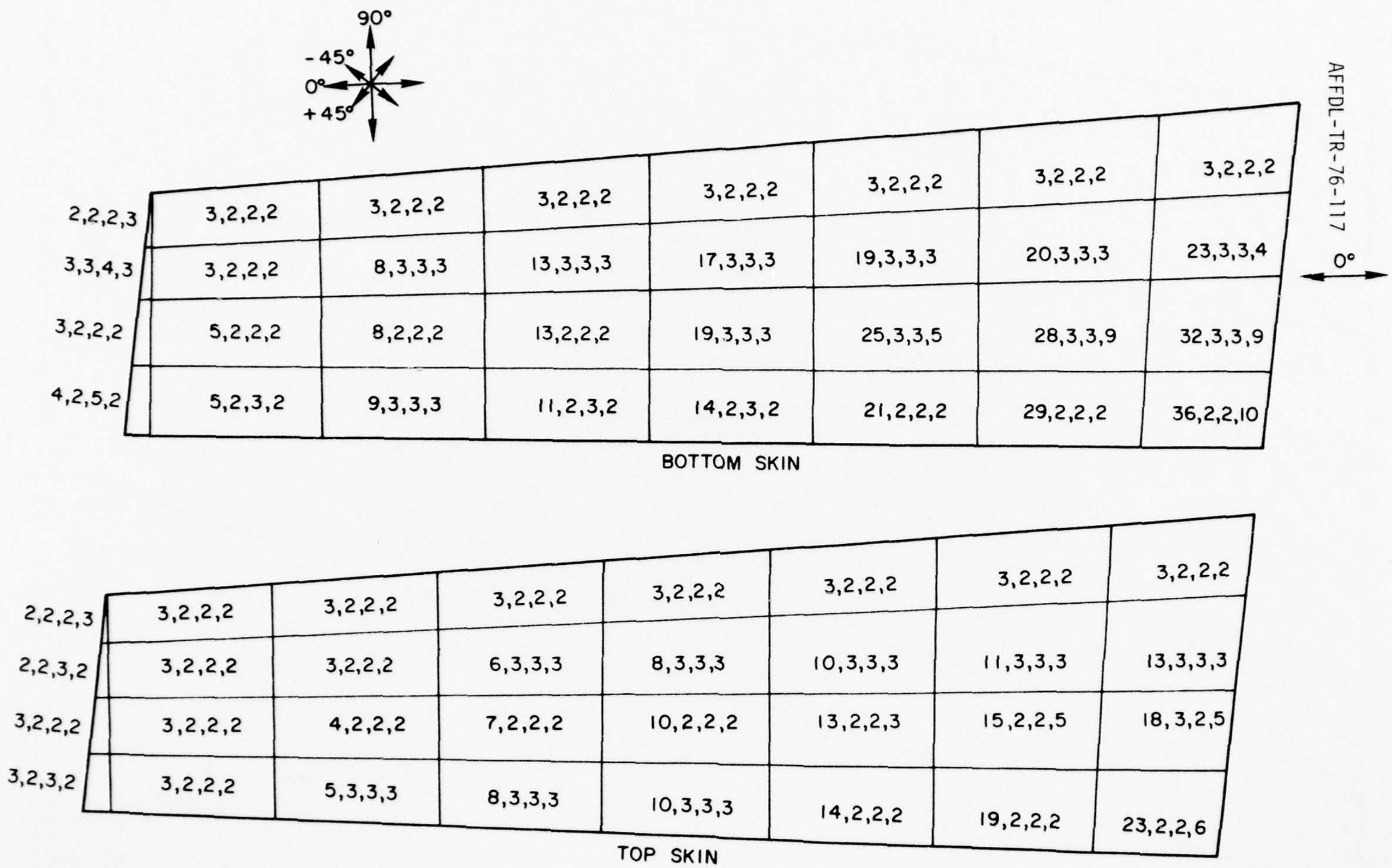


Figure 5. Optimum Design of Wing (Stress Constraint)

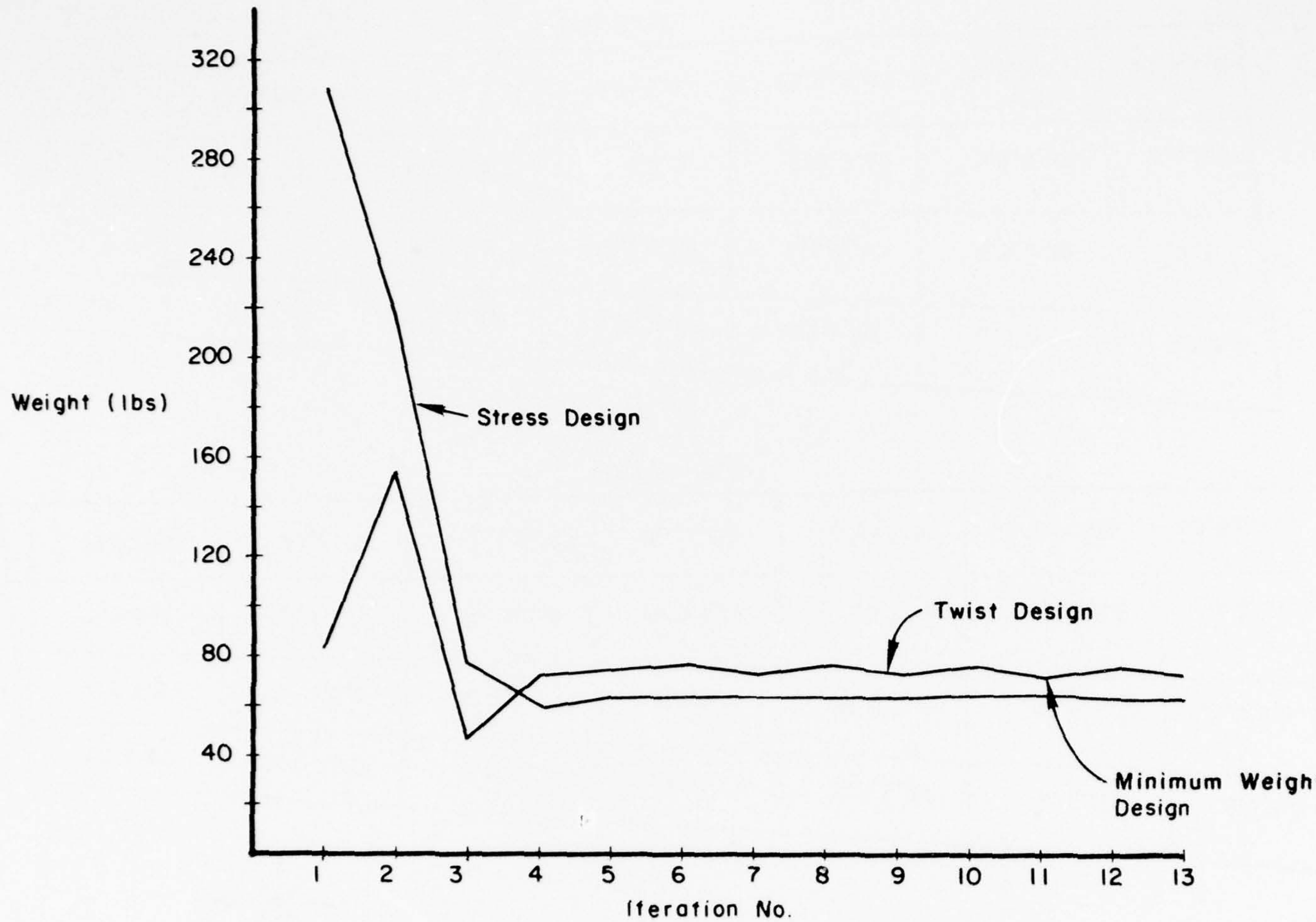


Figure 6. Iteration for Twist Constraint (Wash out -7°)

NUMBER OF LAYERS WITH FIBER ORIENTATIONS IN 0°, 90°, 45°, -45° DIRECTIONS RESPECTIVELY

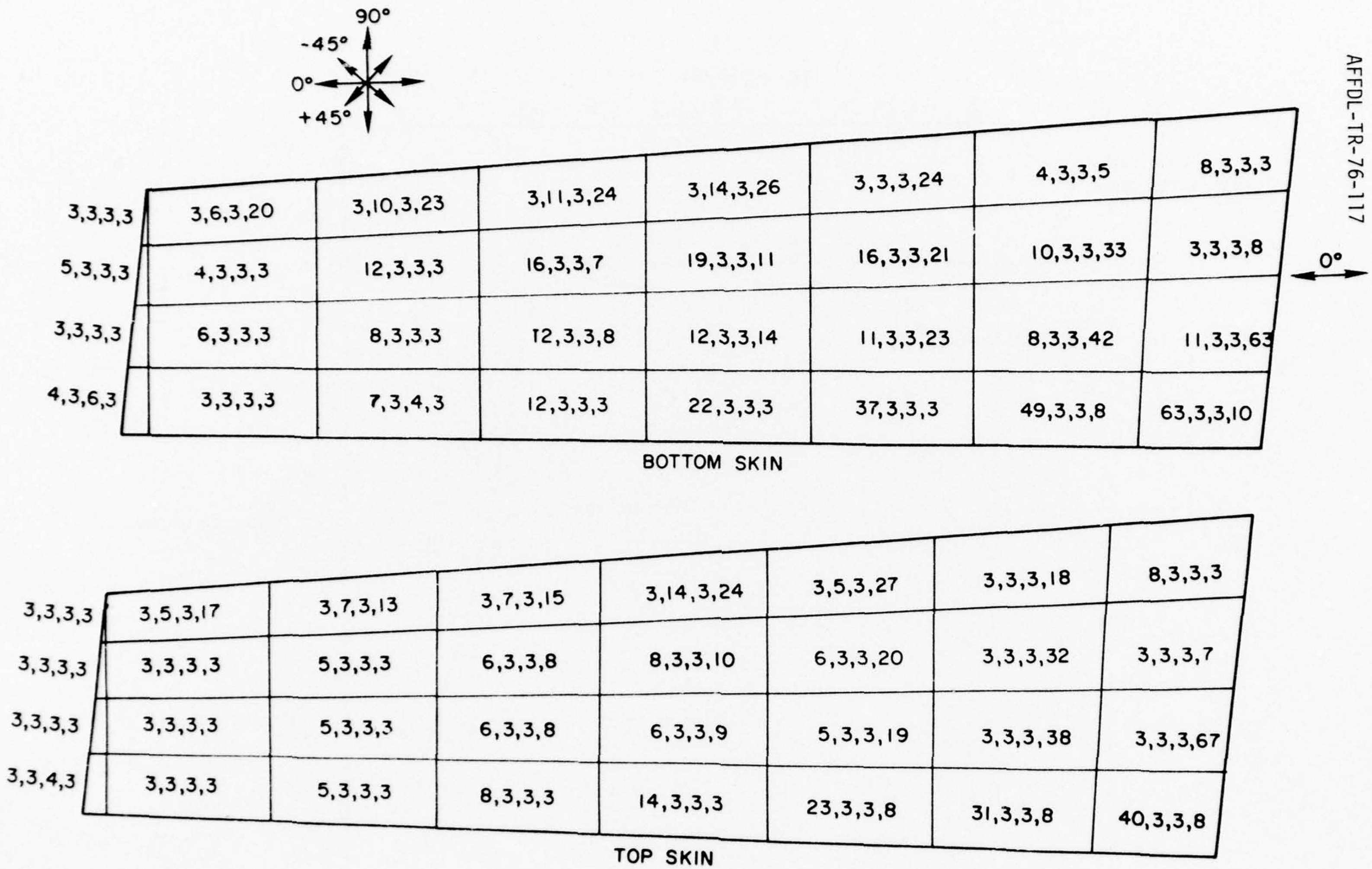


Figure 7. Optimum Design of Wing (Twist Constraint, Wash out -7°)

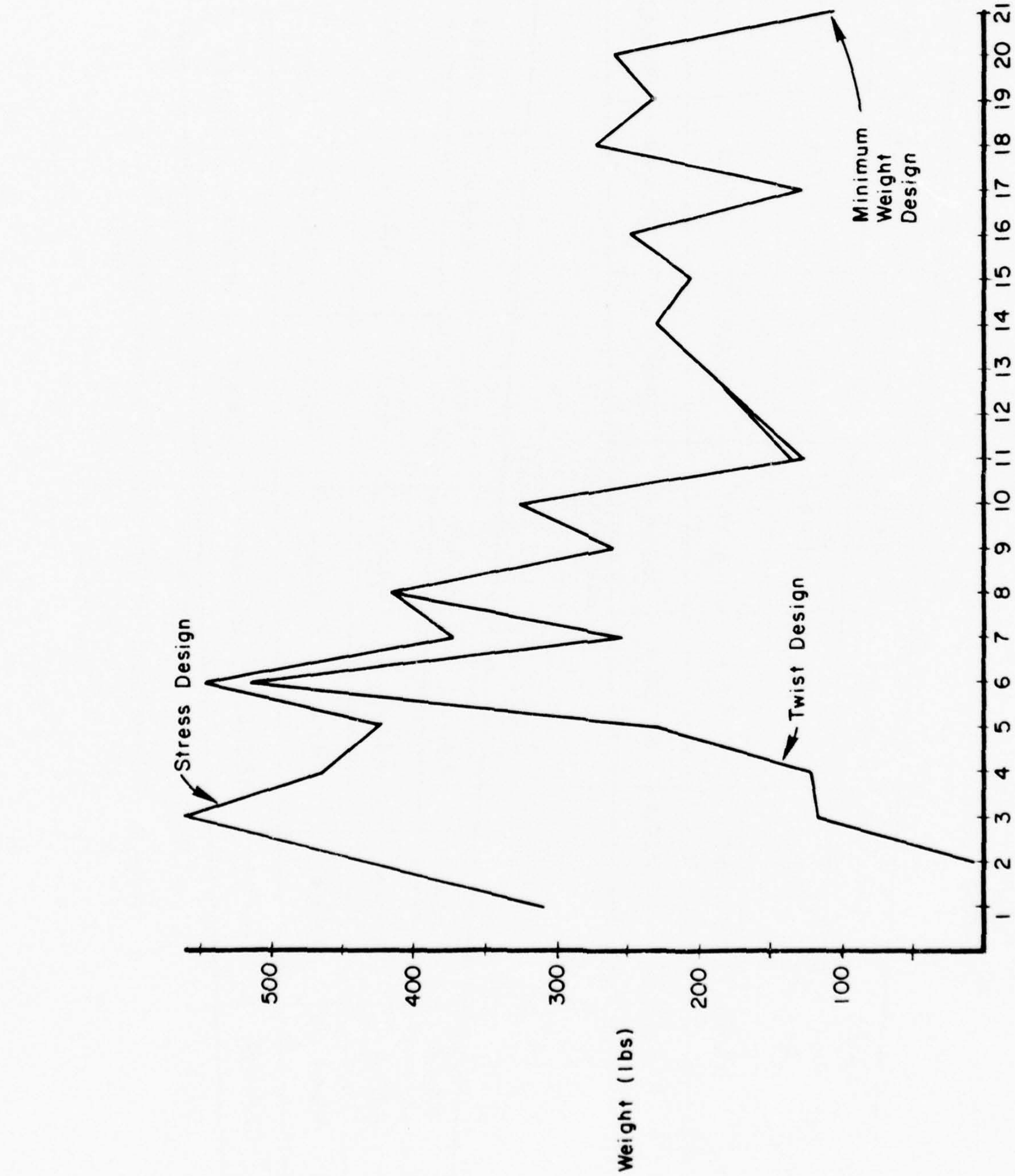


Figure 8. Iteration for Twist Constraint (Wash in 2°)

NUMBER OF LAYERS WITH FIBER ORIENTATIONS IN 0°, 90°, 45°, -45° DIRECTIONS RESPECTIVELY

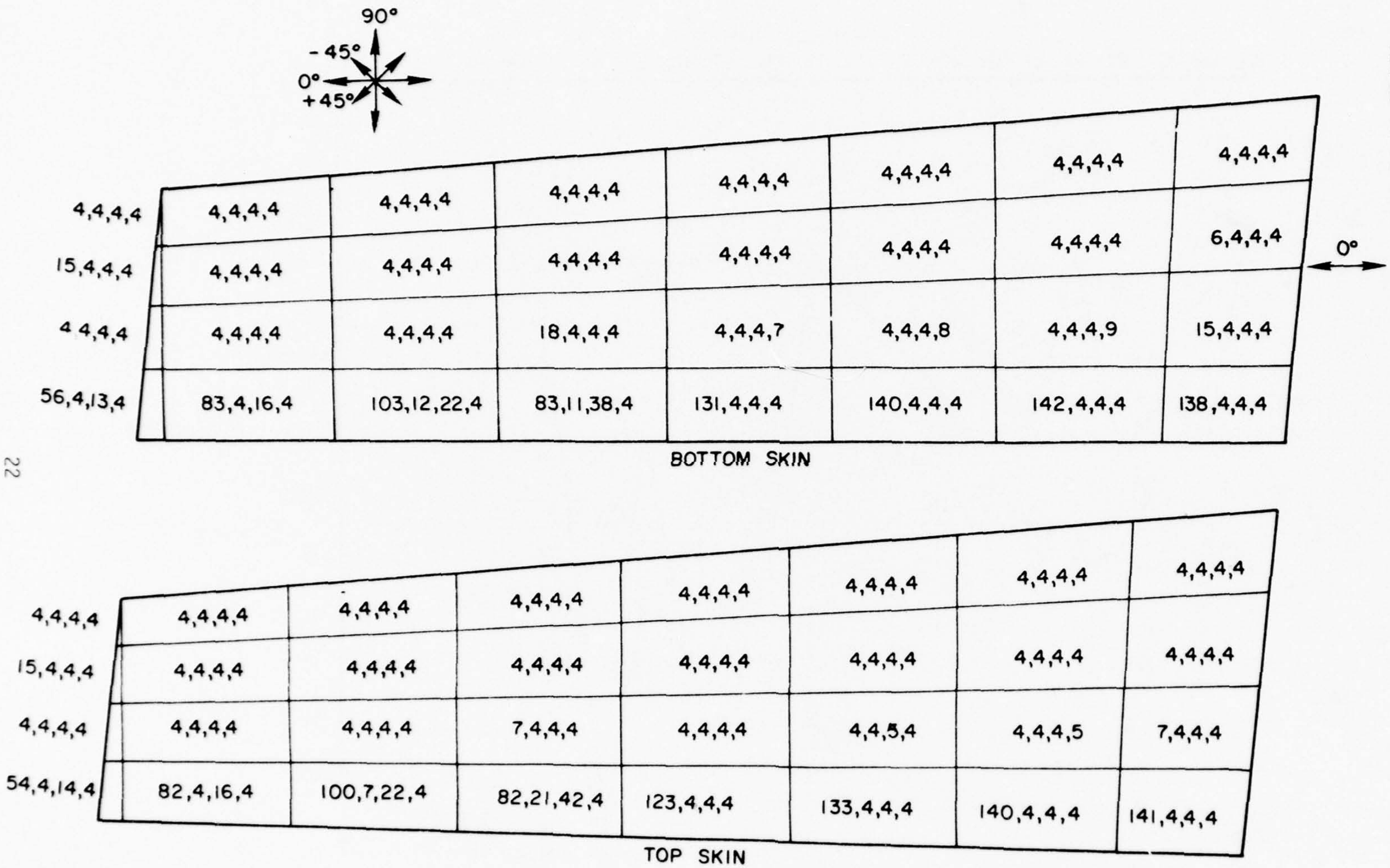


Figure 9. Optimum Design of Wing (Twist Constraint, Wash in 2°)

TABLE 1

ELEMENT CONNECTIONS*

Element No.	A	B	C	D		Element No.	A	B	C	D
1	1	9	11	0		61	73	75	85	83
2	2	10	12	0		62	74	76	86	84
3	1	3	13	11		63	75	77	87	85
4	2	4	14	12		64	76	78	88	86
5	3	5	15	13		65	1	2	10	9
6	4	6	16	14		66	1	2	4	3
7	5	7	17	15		67	3	4	6	5
8	6	8	18	16		68	5	6	8	7
9	9	11	21	19		69	9	10	12	11
10	10	12	22	20		70	11	12	14	13
11	11	13	23	21		71	13	14	16	15
12	12	14	24	22		72	15	16	18	17
13	13	15	25	23		73	19	20	22	21
14	14	16	26	24		74	21	22	24	23
15	15	17	27	25		75	23	24	26	25
16	16	18	28	26		76	25	26	28	27
17	19	21	31	29		77	29	30	32	31
18	20	22	32	30		78	31	32	34	33
19	21	23	33	31		79	33	34	36	35
20	22	24	34	32		80	35	36	38	37
21	23	25	35	33		81	39	40	42	41
22	24	26	36	34		82	41	42	44	43
23	25	27	37	35		83	43	44	46	45
24	26	28	38	36		84	45	46	48	47
25	29	31	41	39		85	49	50	52	51
26	30	32	42	40		86	51	52	54	53
27	31	33	43	41		87	53	54	56	55
28	32	34	44	42		88	55	56	58	57
29	33	35	45	43		89	59	60	62	61
30	34	36	46	44		90	61	62	64	63
31	35	37	47	45		91	63	64	66	65
32	36	38	48	46		92	65	66	68	67
33	39	41	51	49		93	69	70	72	71
34	40	42	52	50		94	71	72	74	73
35	41	43	53	51		95	73	74	76	75
36	42	44	54	52		96	75	76	78	77
37	43	45	55	53		97	9	10	20	19
38	44	46	56	54		98	19	20	30	29
39	45	47	57	55		99	29	30	40	39
40	46	48	58	56		100	39	40	50	49
41	49	51	61	59		101	49	50	60	59
42	50	52	62	60		102	59	60	70	69
43	51	53	63	61		103	69	70	80	79
44	52	54	64	62		104	3	4	14	13
45	53	55	65	63		105	13	14	24	23
46	54	56	66	64		106	23	24	34	33
47	55	57	67	65		107	33	34	44	43
48	56	58	68	66		108	43	44	54	53
49	59	61	71	69		109	53	54	64	63
50	60	62	72	70		110	63	64	74	73
51	61	63	73	71		111	73	74	84	83
52	62	64	74	72		112	7	8	18	17
53	63	65	75	73		113	17	18	28	27
54	64	66	76	74		114	27	28	38	37
55	65	67	77	75		115	37	38	48	47
56	66	68	78	76		116	47	48	58	57
57	69	71	81	79		117	57	58	68	67
58	70	72	82	80		118	67	68	78	77
59	71	73	83	81		119	77	78	88	87
60	72	74	84	82						

*See Figures 3 and 4

Elements 120-158 are posts connecting the top and bottom nodes. For example, element 120 connects nodes 1 and 2 and element 158 connects nodes 77 and 78.

TABLE 2

COORDINATES OF NODES AND APPLIED LOADS

JOINT	-X	-Y	-Z	FORCE-X	FORCE-Y	FORCE-Z
1	70.833	90.000	1.313	0.000	0.000	29.000
2	70.833	90.000	-1.313	0.000	0.000	29.000
3	78.167	90.000	1.500	-2800.000	-6960.000	1130.000
4	78.167	90.000	-1.500	2800.000	6960.000	1130.000
5	85.500	90.000	1.313	0.000	0.000	90.900
6	85.500	90.000	-1.313	0.000	0.000	90.900
7	92.833	90.000	1.125	-9870.000	-9780.000	1130.000
8	92.833	90.000	-1.125	9870.000	9780.000	1130.000
9	63.500	90.000	1.125	235.000	-7380.000	926.000
10	63.500	90.000	-1.125	-235.000	7380.000	926.000
11	69.686	87.471	1.349	0.000	0.000	178.000
12	69.686	87.471	-1.349	0.000	0.000	178.000
13	76.097	84.851	1.586	0.000	0.000	214.000
14	76.097	84.851	-1.586	0.000	0.000	214.000
15	82.746	82.133	1.427	0.000	0.000	253.000
16	82.746	82.133	-1.427	0.000	0.000	253.000
17	89.647	79.312	1.259	-5680.000	2320.000	1020.000
18	89.647	79.312	-1.259	5680.000	-2320.000	1020.000
19	57.266	77.669	1.279	2310.000	-946.000	723.000
20	57.266	77.669	-1.279	-2310.000	946.000	723.000
21	63.992	74.920	1.532	0.000	0.000	314.000
22	63.992	74.920	-1.532	0.000	0.000	314.000
23	70.962	72.071	1.799	0.000	0.000	326.000
24	70.962	72.071	-1.799	0.000	0.000	326.000
25	78.191	69.116	1.617	0.000	0.000	338.000
26	78.191	69.116	-1.617	0.000	0.000	338.000
27	85.692	66.050	1.424	-4070.000	1660.000	902.000
28	85.692	66.050	-1.424	4070.000	-1660.000	902.000
29	51.032	65.339	1.433	1740.000	-713.000	646.000
30	51.032	65.339	-1.433	-1740.000	713.000	646.000
31	58.297	62.369	1.715	0.000	0.000	340.000
32	58.297	62.369	-1.715	0.000	0.000	340.000
33	65.826	59.291	2.012	0.000	0.000	352.000
34	65.826	59.291	-2.012	0.000	0.000	352.000
35	73.635	56.100	1.807	0.000	0.000	365.000
36	73.635	56.100	-1.807	0.000	0.000	365.000
37	81.738	52.787	1.590	-4250.000	1740.000	974.000
38	81.738	52.787	-1.590	4250.000	-1740.000	974.000
39	44.799	53.008	1.587	1820.000	-743.000	694.000
40	44.799	53.008	-1.587	-1820.000	743.000	694.000
41	52.603	49.818	1.898	0.000	0.000	365.000
42	52.603	49.818	-1.898	0.000	0.000	365.000
43	60.691	46.512	2.225	0.000	0.000	378.000
44	60.691	46.512	-2.225	0.000	0.000	378.000
45	69.079	43.083	1.997	0.000	0.000	392.000
46	69.079	43.083	-1.997	0.000	0.000	392.000
47	77.784	39.525	1.756	-4440.000	1820.000	1050.000
48	77.784	39.525	-1.756	4440.000	-1820.000	1050.000
49	38.565	41.678	1.742	1890.000	-773.000	742.000

TABLE 2 (Concluded)

JOINT	-X	-Y	-Z	FORCE-X	FORCE-Y	FORCE-Z
50	38.565	40.678	-1.742	-1890.000	773.000	742.000
51	46.908	37.267	2.082	0.000	0.000	390.000
52	46.908	37.267	-2.082	0.000	0.000	390.000
53	55.555	33.732	2.438	0.000	0.000	404.000
54	55.555	33.732	-2.438	0.000	0.000	404.000
55	64.523	30.067	2.187	0.000	0.000	420.000
56	64.523	30.067	-2.187	0.000	0.000	420.000
57	73.830	26.262	1.922	-4640.000	1900.000	1120.000
58	73.830	26.262	-1.922	4640.000	-1900.000	1120.000
59	32.331	28.347	1.896	2290.000	-937.000	883.000
60	32.331	28.347	-1.896	-2290.000	937.000	883.000
61	41.214	24.716	2.265	0.000	0.000	413.000
62	41.214	24.716	-2.265	0.000	0.000	413.000
63	50.420	20.953	2.651	0.000	0.000	391.000
64	50.420	20.953	-2.651	0.000	0.000	391.000
65	59.967	17.050	2.376	0.000	0.000	368.000
66	59.967	17.050	-2.376	0.000	0.000	368.000
67	69.876	13.000	2.088	-3030.000	1240.000	804.000
68	69.876	13.000	-2.088	3030.000	-1240.000	804.000
69	25.166	14.173	2.073	3070.000	-520.000	1040.000
70	25.166	14.173	-2.073	-3070.000	520.000	1040.000
71	35.583	12.304	2.446	0.000	0.000	433.000
72	35.583	12.304	-2.446	0.000	0.000	433.000
73	46.181	10.403	2.827	0.000	0.000	370.000
74	46.181	10.403	-2.827	0.000	0.000	370.000
75	56.964	8.469	2.502	0.000	0.000	304.000
76	56.964	8.469	-2.502	0.000	0.000	304.000
77	67.938	6.500	2.169	-1370.000	262.000	446.000
78	67.938	6.500	-2.169	1370.000	-262.000	446.000
79	18.000	0.000	2.250	1730.000	0.000	544.000
80	18.000	0.000	-2.250	-1730.000	0.000	544.000
81	30.000	0.000	2.625	0.000	0.000	224.000
82	30.000	0.000	-2.625	0.000	0.000	224.000
83	42.000	0.000	3.000	0.000	0.000	190.000
84	42.000	0.000	-3.000	0.000	0.000	190.000
85	54.000	0.000	2.625	0.000	0.000	155.000
86	54.000	0.000	-2.625	0.000	0.000	155.000
87	66.000	0.000	2.250	-715.000	0.000	226.000
88	66.000	0.000	-2.250	715.000	0.000	226.000

TABLE 3

PROPERTIES AND ALLOWABLE STRESSES

Properties	MATERIAL: Graphite Epoxy	MATERIAL: Aluminum
E_{11} (psi)	18.5×10^6	10.5×10^6
E_{22} (psi)	1.6×10^6	10.5×10^6
Poisson's Ratio	0.25	0.3
Shear Modulus (psi)	0.65×10^6	4.038×10^6
Specific Weight (lbs/in^3)	0.055	0.1
Lamina Thickness(in)	0.0052	—
Allowable Stresses (Ksi)		
F_x (tension)	139.0	45.0
F_x (compression)	86.0	45.0
F_y (tension)	—	45.0
F_y (compression)	—	45.0
F_{xy}	46.8	25.9

TABLE 4

ITERATIVE HISTORY FOR THE STRESS CONSTRAINT PROBLEM

Iteration No.	Weight (lbs)	Iteration No.	Weight (lbs)
1	312.84	9	45.71
2	112.18	10	45.59
3	68.37	11	45.49
4	59.89	12	45.49
5	55.78	13	45.51
6	47.16	14	45.45
7	45.63	15	45.45
8	45.75		

TABLE 5

ELEMENT AND LAYER THICKNESSES FOR THE MINIMUM WEIGHT STRESS CONSTRAINT DESIGN

	Thickness (in.)				
Element No.	Total	0°	90°	-45°	+45°
1	.0418	.0104	.0104	.0106	.0104
2	.0418	.0104	.0104	.0106	.0104
3	.0484	.0108	.0108	.0159	.0108
4	.0420	.0104	.0104	.0108	.0104
5	.0419	.0107	.0104	.0104	.0104
6	.0419	.0107	.0104	.0104	.0104
7	.0643	.0189	.0104	.0246	.0104
8	.0477	.0117	.0104	.0152	.0104
9	.0420	.0108	.0104	.0104	.0104
10	.0470	.0108	.0104	.0104	.0104
11	.0430	.0118	.0104	.0104	.0104
12	.0419	.0107	.0104	.0104	.0104
13	.0563	.0251	.0104	.0104	.0104
14	.0425	.0113	.0104	.0104	.0104
15	.0548	.0216	.0104	.0124	.0104
16	.0426	.0114	.0104	.0104	.0104
17	.0423	.0111	.0104	.0104	.0104
18	.0421	.0109	.0104	.0104	.0104
19	.0723	.0387	.0112	.0112	.0112
20	.0465	.0153	.0104	.0104	.0104
21	.0697	.0385	.0104	.0104	.0104
22	.0507	.0195	.0104	.0104	.0104
23	.0765	.0419	.0106	.0135	.0106
24	.0576	.0247	.0110	.0110	.0110
25	.0426	.0114	.0104	.0104	.0104
26	.0422	.0110	.0104	.0104	.0104
27	.0974	.0628	.0115	.0115	.0115
28	.0598	.0285	.0104	.0104	.0104
29	.0971	.0659	.0104	.0104	.0104
30	.0641	.0329	.0104	.0104	.0104
31	.0908	.0566	.0104	.0135	.0104
32	.0719	.0386	.0111	.0111	.0111
33	.0426	.0114	.0104	.0104	.0104
34	.0422	.0110	.0104	.0104	.0104
35	.1180	.0831	.0116	.0116	.0116
36	.0725	.0409	.0105	.0105	.0105
37	.1320	.0978	.0106	.0106	.0130
38	.0806	.0494	.0104	.0104	.0104
39	.1040	.0716	.0104	.0117	.0104
40	.0829	.0508	.0107	.0107	.0107
41	.0425	.0113	.0104	.0104	.0104
42	.0422	.0110	.0104	.0104	.0104
43	.1340	.0987	.0118	.0118	.0118
44	.0822	.0503	.0106	.0106	.0106
45	.1710	.1260	.0106	.0106	.0236

TABLE 5 (Concluded)

Element No.	Thickness (in.) Total	0°	90°	-45°	+45°
46	.0999	.0665	.0104	.0104	.0126
47	.1390	.1078	.0104	.0104	.0104
48	.1010	.0697	.0104	.0104	.0104
49	.0424	.0112	.0104	.0104	.0104
50	.0422	.0110	.0104	.0104	.0104
51	.1360	.1017	.0115	.0115	.0115
52	.0855	.0537	.0106	.0106	.0106
53	.2100	.1431	.0107	.0107	.0456
54	.1230	.0774	.0104	.0104	.0249
55	.1810	.1498	.0104	.0104	.0104
56	.1270	.0958	.0104	.0104	.0104
57	.0421	.0109	.0104	.0104	.0104
58	.0421	.0109	.0104	.0104	.0104
59	.1580	.1192	.0109	.0109	.0170
60	.0969	.0655	.0105	.0105	.0105
61	.2300	.1645	.0107	.0107	.0441
62	.1360	.0907	.0104	.0104	.0245
63	.2510	.1828	.0104	.0104	.0474
64	.1690	.1175	.0104	.0104	.0307

TABLE 6
SHEAR PANEL AND BAR DIMENSIONS FOR THE
MINIMUM WEIGHT STRESS CONSTRAINT DESIGN

Shear Panels		Shear Panels	
Element No.	Thickness (in.)	Element No.	Thickness (in.)
65-104	.0416	112	.0416
105	.0446	113	.0515
106	.0658	114	.0570
107	.0827	115	.0637
108	.0900	116	.0772
109	.0926	117	.0973
110	.0992	118	.1210
111	.1090	119	.1390

Bars	
Element No.	Area (in^2)
120-158	.0416

TABLE 7
DEFLECTIONS AT TIP FOR THE
MINIMUM WEIGHT STRESS CONSTRAINT DESIGN

Node 7 (trailing edge) Direction (in)			Node 9 (leading edge) Direction (in)			Twist (deg.)
X	Y	Z	X	Y	Z	
-.0565	-.1961	13.4306	-.0435	-.1632	10.6296	-5.458

TABLE 8

ITERATIVE HISTORY FOR THE TWIST CONSTRAINT PROBLEM (WASH OUT 7°)

Cycle No.	Weight (lbs.)		Twist (deg.)	
	Stress Design	Twist Design	Stress Design	Twist Design
1	309.34	82.70	-1.879	-7.000
2	213.96	153.70	-5.040	-7.000
3	76.61	46.31	-4.244	-7.000
4	58.72	72.22	-8.589	-7.000
5	63.49	73.87	-8.130	-7.000
6	63.89	76.83	-8.400	-7.000
7	63.78	72.70	-7.967	-7.000
8	63.71	76.82	-8.421	-7.000
9	62.95	72.93	-8.097	-7.000
10	64.56	76.51	-8.280	-7.000
11	64.28	71.86	-7.815	-7.000
12	63.40	76.07	-8.381	-7.000
13	63.53	72.35	-7.959	-7.000

TABLE 9

DEFLECTIONS AT TIP FOR THE MINIMUM WEIGHT TWIST CONSTRAINT DESIGN (WASH OUT 7°)

Node 7 (trailing edge) Direction (in)			Node 9 (leading edge) Direction (in)			Twist (deg.)
X	Y	Z	X	Y	Z	
-.0995	-.1594	11.6800	-.0881	-.1073	8.0799	-7.00

TABLE 10

ELEMENT AND LAYER THICKNESSES FOR THE MINIMUM WEIGHT TWIST CONSTRAINT DESIGN (WASH OUT 7°)

	Thickness (in.)				
Element No.	Total	0°	90°	-45°	+45°
1	.0494	.0124	.0124	.0124	.0124
2	.0494	.0124	.0124	.0124	.0124
3	.0579	.0208	.0124	.0124	.0124
4	.0494	.0124	.0124	.0124	.0124
5	.0494	.0124	.0124	.0124	.0124
6	.0494	.0124	.0124	.0124	.0124
7	.0736	.0191	.0123	.0298	.0123
8	.0554	.0123	.0123	.0184	.0123
9	.1550	.0123	.0302	.0123	.1001
10	.1360	.0124	.0250	.0124	.0863
11	.0572	.0201	.0124	.0124	.0124
12	.0494	.0124	.0124	.0124	.0124
13	.0678	.0307	.0124	.0124	.0124
14	.0494	.0124	.0124	.0124	.0124
15	.0530	.0153	.0124	.0130	.0124
16	.0494	.0124	.0124	.0124	.0124
17	.1940	.0124	.0513	.0124	.1179
18	.1250	.0124	.0338	.0124	.0665
19	.0988	.0617	.0124	.0124	.0124
20	.0590	.0220	.0123	.0123	.0123
21	.0768	.0397	.0124	.0124	.0124
22	.0581	.0210	.0124	.0124	.0124
23	.0769	.0347	.0124	.0175	.0124
24	.0595	.0225	.0123	.0123	.0123
25	.2020	.0124	.0538	.0124	.1234
26	.1330	.0124	.0341	.0124	.0742
27	.1400	.0818	.0123	.0123	.0335
28	.0937	.0298	.0124	.0124	.0392
29	.1240	.0581	.0124	.0124	.0411
30	.0947	.0290	.0124	.0124	.0410
31	.0962	.0591	.0124	.0124	.0124
32	.0769	.0398	.0124	.0124	.0124
33	.2290	.0123	.0699	.0124	.1344
34	.2190	.0124	.0719	.0124	.1224
35	.1750	.0947	.0123	.0123	.0551
36	.1110	.0381	.0124	.0123	.0482
37	.1560	.0610	.0123	.0123	.0703
38	.1020	.0309	.0124	.0124	.0464
39	.1500	.1129	.0124	.0124	.0124
40	.1090	.0719	.0124	.0124	.0124
41	.1580	.0124	.0124	.0124	.1209
42	.1830	.0124	.0209	.0124	.1373
43	.2120	.0823	.0123	.0123	.1051
44	.1560	.0289	.0124	.0124	.1024
45	.1960	.0536	.0123	.0123	.1177

TABLE 10 (Concluded)

	Thickness (in.)				
Element No.	Total	0°	90°	-45°	+45°
46	.1480	.0251	.0123	.0123	.0982
47	.2250	.1879	.0124	.0124	.0124
48	.1810	.1152	.0124	.0124	.0410
49	.0678	.0185	.0123	.0123	.0246
50	.0780	.0123	.0123	.0123	.0410
51	.2460	.0499	.0123	.0123	.1714
52	.2020	.0123	.0123	.0123	.1650
53	.2830	.0415	.0124	.0219	.2162
54	.2300	.0124	.0124	.0124	.1930
55	.3200	.2543	.0124	.0124	.0410
56	.2270	.1612	.0124	.0124	.0411
57	.0780	.0410	.0123	.0123	.0123
58	.0780	.0410	.0123	.0123	.0123
59	.0759	.0124	.0124	.0124	.0388
60	.0704	.0123	.0123	.0123	.0334
61	.4040	.0565	.0124	.0123	.3228
62	.3800	.0124	.0124	.0124	.3430
63	.3970	.3252	.0123	.0123	.0472
64	.2700	.2044	.0123	.0123	.0409

TABLE 11

SHEAR PANEL AND BAR DIMENSIONS FOR THE MINIMUM WEIGHT TWIST CONSTRAINT DESIGN (WASH OUT 7°)

Shear Panels		Shear Panels	
Element No.	Thickness (in.)	Element No.	Thickness (in.)
65-68	.0485	96	.0485
69	.0511	97	.1380
70	.0434	98	.1370
71-74	.0433	99	.1490
75	.0485	100	.1450
76	.0433	101	.0898
77	.0433	102-105	.0485
78-81	.0485	106	.0605
82	.0433	107	.0552
83	.0485	108	.0485
84	.0485	109	.0655
85	.0433	110	.1340
86	.0433	111	.1950
87	.0485	112	.0485
88	.0485	113	.0592
89	.0618	114	.0681
90	.0472	115	.0968
91	.0485	116	.1340
92	.0485	117	.1690
93	.0433	118	.2070
94	.0433	119	.2290
95	.0485		

Bars		Bars	
Node No.	Area (in^2)	Node No.	Area (in^2)
120-122	.0485	142	.0433
123	.0433	143	.0485
124	.0485	144	.0485
125	.0433	145-147	.0433
126	.0433	148	.0485
127	.0485	149	.0433
128	.0485	150	.0485
129	.0433	151	.0433
130	.0433	152	.0485
131	.0485	153	.0433
132	.0433	154	.0433
133-137	.0485	155	.0485
138-140	.0433	156-158	.0433
141	.0485		

TABLE 12

ITERATIVE HISTORY FOR THE TWIST CONSTRAINT PROBLEM (WASH IN 2°)

Cycle No.	Weight (lbs.)		Twist (deg.)	
	Stress Design	Twist Design	Stress Design	Twist Design
1	309.34		-1.879	
2	434.58	8.24	.0379	2.000
3	558.98	116.80	.418	2.000
4	461.96	122.51	.531	2.000
5	422.68	230.15	1.090	2.000
6	546.74	515.48	1.886	2.000
7	370.82	253.58	1.368	2.000
8	416.46	416.46	2.000	2.000
9	258.97	258.97	2.000	2.000
10	326.38	326.38	2.000	2.000
11	135.14	126.67	1.875	2.000
12	164.13	164.13	2.000	2.000
13	194.59	194.59	2.000	2.000
14	230.57	230.57	2.000	2.000
15	206.63	206.63	2.000	2.000
16	248.42	248.42	2.000	2.000
17	127.60	127.60	2.000	2.000
18	274.28	274.28	2.000	2.000
19	232.19	232.19	2.000	2.000
20	260.97	260.97	2.000	2.000
21	105.54	105.54	2.000	2.000

TABLE 13

DEFLECTIONS AT TIP FOR THE MINIMUM WEIGHT
TWIST CONSTRAINT DESIGN (WASH IN 2°)

Node 7 (trailing edge) Direction (in)			Node 9 (leading edge) Direction (in)			Twist (deg.)
X	Y	Z	X	Y	Z	
.0607	-.1073	4.4931	.0449	-.1172	5.5179	2.000

TABLE 14

ELEMENT AND LAYER THICKNESSESS FOR THE MINIMUM WEIGHT TWIST CONSTRAINT DESIGN (WASH IN 2°)

	Thickness (in.)				
Element No.	Total	0°	90°	-45°	+45°
1	.0680	.0170	.0170	.0170	.0170
2	.0680	.0170	.0170	.0170	.0170
3	.1290	.0778	.0171	.0171	.0171
4	.1280	.0769	.0170	.0170	.0170
5	.0680	.0170	.0170	.0170	.0170
6	.0708	.0170	.0170	.0197	.0170
7	.3880	.2877	.0170	.0638	.0194
8	.3870	.2807	.0170	.0723	.0170
9	.0680	.0170	.0170	.0170	.0170
10	.0680	.0170	.0170	.0170	.0170
11	.0680	.0170	.0170	.0170	.0170
12	.0680	.0170	.0170	.0170	.0170
13	.0695	.0170	.0170	.0170	.0185
14	.0680	.0170	.0170	.0170	.0170
15	.5460	.4290	.0170	.0829	.0170
16	.5400	.4253	.0170	.0806	.0170
17	.0680	.0170	.0170	.0170	.0170
18	.0680	.0170	.0170	.0170	.0170
19	.0680	.0170	.0170	.0170	.0170
20	.0680	.0170	.0170	.0170	.0170
21	.0695	.0170	.0170	.0170	.0185
22	.0680	.0170	.0170	.0170	.0170
23	.7190	.5308	.0611	.1102	.0170
24	.6830	.5167	.0352	.1141	.0170
25	.0680	.0170	.0170	.0170	.0170
26	.0680	.0170	.0170	.0170	.0170
27	.0680	.0170	.0170	.0170	.0170
28	.0680	.0170	.0170	.0170	.0170
29	.1420	.0905	.0170	.0170	.0174
30	.0828	.0318	.0170	.0170	.0170
31	.6940	.4272	.0561	.1936	.0170
32	.7660	.4265	.1092	.2133	.0170
33	.0680	.0170	.0170	.0170	.0170
34	.0680	.0170	.0170	.0170	.0170
35	.0680	.0170	.0170	.0170	.0170
36	.0680	.0170	.0170	.0170	.0170
37	.0850	.0170	.0170	.0170	.0339
38	.0692	.0170	.0170	.0182	.0170
39	.7290	.6780	.0170	.0170	.0170
40	.6900	.6390	.0170	.0170	.0170
41	.0680	.0170	.0170	.0170	.0170
42	.0680	.0170	.0170	.0170	.0170
43	.0680	.0170	.0170	.0170	.0170
44	.0680	.0170	.0170	.0170	.0170
45	.0919	.0170	.0170	.0170	.0408

TABLE 14 (Concluded)

Element No.	Thickness (in.) Total	0°	90°	-45°	+45°
46	.0767	.0170	.0170	.0256	.0170
47	.7790	.7280	.0170	.0170	.0170
48	.7400	.6889	.0170	.0170	.0170
49	.0680	.0170	.0170	.0170	.0170
50	.0680	.0170	.0170	.0170	.0170
51	.0709	.0199	.0170	.0170	.0170
52	.0680	.0170	.0170	.0170	.0170
53	.0927	.0170	.0170	.0170	.0416
54	.7290	.1701	.1701	.1701	.2188
55	.7870	.7359	.0170	.0170	.0170
56	.7780	.7270	.0170	.0170	.0170
57	.0680	.0170	.0170	.0170	.0170
58	.0680	.0170	.0170	.0170	.0170
59	.0807	.0297	.0170	.0170	.0170
60	.0680	.0170	.0170	.0170	.0170
61	.1300	.0755	.0170	.0170	.0206
62	.0852	.0342	.0170	.0170	.0170
63	.7650	.7140	.0170	.0170	.0170
64	.7800	.7289	.0170	.0170	.0170

TABLE 15

SHEAR PANEL AND BAR DIMENSIONS FOR THE MINIMUM WEIGHT TWIST CONSTRAINT DESIGN (WASH IN 2°)

Shear Panels		Shear Panels	
Element No.	Thickness (in.)	Element No.	Thickness (in.)
65-72	.0680	110	.1090
73	.0417	111	.1460
74	.0680	112	.1090
75	.0417	113	.1430
76-80	.0680	114	.1330
81-86	.0417	115	.1410
87-91	.0680	116	.1810
92	.0417	117	.1910
93-108	.0680	118	.1330
109	.0743	119	.1570

Bars		Bars	
Element No.	Area (in^2)	Element No.	Area (in^2)
120	.0680	136	.0680
121	.0417	137	.0680
122-124	.0680	138	.0417
125-129	.0417	139-141	.0680
130	.0680	142	.0417
131	.0680	143	.0417
132	.0417	144-156	.0680
133	.0417	157	.0417
134	.0680	158	.0680
135	.0417		

REFERENCES

1. Venkayya, V. B., Khot, N. S., and Berke, L., "Application of Optimality Criteria Approaches to Automated Design of Large Practical Structures," presented at the Second Symposium on Structural Optimization, Milan, Italy, 2-4 April 1973. AGARD Conference Proceedings No. 123.

2. Berke, L., Khot, N. S., "Use of Optimality Criteria Methods for Large Scale Systems," AGARD Lecture Series No. 70 on Structural Optimization, 1973.

3. Khot, N. S., Venkayya, V. B., Berke, L., Optimum Design of Composite Structures with Stress and Displacement Constraints, presented at AIAA 13th Aerospace Sciences Meeting, Pasadena, California, January 20-22, 1975 (AIAA Paper 75-141).

☆ U. S. GOVERNMENT PRINTING OFFICE: 1977 — 757-001/263